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OPTIMAL CONTROL OF INGOT HEATING IN FLOW-THROUGH FURNACES

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The algorithm developed here allows the ingot-heating conditions in a furnace corresponding to the least skin formation to be determined. The influence of the furnace atmosphere on skin growth in heating is investigated.

Economic efficiency – in particular, the choice of optimal technological conditions with respect to specified criteria – is one of the most important questions in studying heating-furnace operation. In considering the optimal control of ingot heating, effective algorithms have been developed for linear problems [1]. For nonlinear problems, it is difficult to obtain a control algorithm. However, the introduction of nonlinear components in the mathematical description of the process permits the construction of a more adequate mathematical model. Consider the nonlinear problem of skin minimization for bodies with a small internal resistance

$$\frac{dT}{dt} = k'\left(\alpha\left(T_c(t) - T(t)\right) + \sigma\left(T_c^4(t) - T^4(t)\right)\right), \tag{1}$$

$$T(0) = T_0, \quad T(t_f) = T_f, \tag{2}$$

$$\int_0^{t_f} \frac{\varkappa}{T(t)} \exp\left[-\beta/T(t)\right] dt \to \min_{T_c(t)}. \tag{3}$$

The optimal-control problem consists of the choice of conditions of temperature variation of the medium over time such as to ensure ingot heating from a specified initial temperature T_0 to a specified final temperature T_f, with minimum skin formation at time t_f. The control function $T_c(t)$ satisfies the constraint $T_0 < A_1 \le T_c(t) \le T_f \le A_2$ for any $0 \le t \le t_f$ and is a piecewise-continuous function of t with a finite number of points of discontinuity.

Using the mathematical apparatus of [1, 2], the optimal heating trajectory of the metal, the optimal control, and the switching time of the controlling action (temperature of the heating medium) are obtained. The procedure for elucidating the structure of the controlling function, the optimal trajectory, and the algorithm for solving Eqs. (1)-(3) may be found in the Appendix.

After finding the switching time t_2, the optimal control is written in the form

$$T_c(t) = \begin{cases} A_1, & 0 \le t < t_2, \\ A_2, & t_2 \le t \le t_f. \end{cases}$$

Thus, a two-stage heating graph of fine-grade ingots is obtained, as well as an algorithm for determining the switching time.

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Industrial tests confirm the efficiency of the algorithm for solving the problem of nonlinear control of ingot heating by major optimization.

Two industrial experiments are conducted on the 320/150 furnace of the Belorussian Metallurgical Plant. In experiment A, the parameters characterizing the oxidation dynamics κ_A, β_A are determined by the plant technology. The least-squares method gives the coefficient values: $\kappa_A = 0.418528 \cdot 10^7$, $\beta_A = 0.9 \cdot 10^4$. The ingot is heated from 50 to 1188°C in 100 min. The skin calculated in the case of heating by the plant technology is 3.09161 kg/m^2. The algorithm to find the switching time indicates that the best (mean) temperature of the medium is 700°C at $0 \le t \le 75$ min, and 1200°C at $75 \le t \le 100$ min. Calculations show that the ingot is heated from 50°C to 1185°C and the least possible amount of skin in the whole process is 1.38191 kg/m^2.

Numerical experiment shows that significant decrease in the skin is achieved.

The dynamics of skin growth is now changed: in experiment B, the parameters of the furnace atmosphere are characterized by a higher (by 0.15) gas-air ratio in the first three zones of the furnace and a lower (by 0.09) ratio in the other zones than in experiment A. The coefficients $\kappa_B = 0.141049 \cdot 10^5$ and $\beta_B = 3 \cdot 10^3$ are obtained.

In 160 min, the ingot is heated from 50 to 1110°C. At the end of the heating, the skin amounts to 1.3876 kg/m^2. The less stringent heating conditions in experiment B permit significant reduction in the rate of oxidation. The algorithm for finding the switching time indicates that the best (mean) temperature of the medium is 680°C at $0 \le t < 146.3$ min and 1200°C at $146.3 \le t \le 160$ min; the ingot is heated from 50 to 1111°C here, and the minimal skin formed in the process is 1.14615 kg/m^2.

Note also that, after optimizing the temperature of the medium, the amount of skin formed in the heating time may be reduced simply by changing the furnace atmosphere (the temperature conditions remain the same). Thus, on choosing the optimal temperature conditions for the initial data in experiment A and then changing the atmosphere in the furnace to correspond to experiment B, the skin formed in the heating process (100 min) is 0.741753 kg/m^2, which is 0.6 kg/m^2 less than for the same temperature conditions without changing the atmosphere.

Thus, the theoretical and experimental data show that the most effective method of skin reduction is to create a less oxidative atmosphere in the furnace. Without changing the furnace atmosphere, the basic resource in terms of skin reduction is to develop optimal temperature conditions.

APPENDIX

In view of Corollary 1 of [1], the control $u_\omega(t)$ realizing an infinite optimal trajectory (IOT) must be a piecewise-constant function.

It is simple to show that the IOT $\omega(t)$ is found as a result of integrating Eq. (1) in the case where $T_c(t) = A_1$, $t > 0$, $T(0) = T_0$, i.e., $u_\omega(t) = A_1$, $t > 0$, and condition (b) of the theorem formulated and proven in [1] is satisfied for the given IOT.

Calculating the derivative with respect to T for the integrand in Eq. (3), it is found that the integrand decreases with decrease in temperature of the metal.

Since, for any arc of the trajectory $\omega(t)$ and any $0 \le t_1 < t_2$, there is a unique solution of Eq. (1) passing through the points $\omega(t_1)$ and $\omega(t_2)$, then $\omega(t)$ is the IOT for the problem in Eqs. (1)-(3) by definition [1]. Since, by construction, $T(t) \ge \omega(t)$, $\forall t > 0$ for any permissible process $T(t)$, $T_c(t)$ of the problem in Eqs. (1)-(3), condition (b) of the theorem of [1] is satisfied.

Therefore, it is obvious that the solution of the problem with a free right-hand end of the trajectory $\varphi_1(t)$ [1] coincides with $\omega(t)$, $t > 0$.

The solution of the problem with a free left-hand end of the trajectory $\varphi_2(t)$ and time t_2 is now found.

Since $\lim_{t \to \infty} \omega(t) = A_1$, there is a time $t_2 < t_f$ such that Eq. (1) with the boundary conditions $T(t_2) = \omega(t_2)$, $T(t_f) = T_f$ on the segment $[t_2, t_f]$ has a solution $\Psi_2(t)$ but the analogous boundary problem at $t_2^1 > t_2$ has no solution in all possible conditions of temperature variation of the medium.

The trajectory $\varphi_2(t)$ may be obtained as the solution of the following problem

$$\frac{d\varphi_2}{dt} = k'(\alpha(A_2 - \varphi_2(t)) + \sigma(A_2^4 - \varphi_2^4(t))), \tag{4}$$

$$t_2 \leqslant t \leqslant t_f,$$
$$\varphi_2(t_2) = \omega(t_2), \quad \varphi_2(t_f) = T_f. \tag{5}$$

It may be shown that $\varphi_2(t)$, $t_2 \leq t \leq t_f$ and t_2 are the desired trajectory and time. Let $x_2(t)$ be the optimal solution of the corresponding problem with a free left-hand end of the trajectory on the segment $[t_2, t_f]$, when $x_2(t_2) > \varphi_2(t_2)$, since otherwise this contradicts the choice of t_2. In view of the continuity of $x_2(t)$ and $\varphi_2(t)$ on segment $[t_2, t_f]$ and Eqs. (4) and (5), it follows that there is a time $t' < t_f$ such that: $x_2(t') = \varphi_2(t')$. It is obvious that: $x_2(t) = \varphi_2(t)$, $t_f \geq t \geq t'$ and $x_2(t) > \varphi_2(t)$, $t' > t \geq t_2$. Hence

$$\int_{t_2}^{t_f} \frac{\varkappa}{\varphi_2(t)} \exp[-\beta/\varphi_2(t)]\, dt < \int_{t_2}^{t_f} \frac{\varkappa}{x_2(t)} \exp[-\beta/x_2(t)]\, dt,$$

i.e., $x_2(t)$ is not the optimal solution of the problem with a free left-hand end of the trajectory on the segment $[t_2, t_f]$. This is a contradiction. Therefore, $\varphi_2(t)$ is an optimal solution of the problem with a free left-hand end of the trajectory.

Thus, the algorithm for solving Eqs. (1)-(3) consists in determining t_2, which is found as the only root of the equation

$$\omega(\tau) - \varphi_2(\tau) = 0, \tag{6}$$

where $\omega(\tau)$ and $\varphi_2(\tau)$ are established as a result of solving the problems

$$\frac{d\omega}{dt} = k'(\alpha(A_1 - \omega(t)) + \sigma(A_1^4 - \omega^4(t))), \quad 0 \leqslant t \leqslant \tau, \quad \tau > 0, \tag{7}$$

$$\omega(0) = T_0; \tag{8}$$

$$\frac{d\varphi_2}{dt} = k'(\alpha(A_2 - \varphi_2(t)) + \sigma(A_2^4 - \varphi_2^4(t))), \quad \tau \leqslant t \leqslant t_f, \tag{9}$$

$$\varphi_2(\tau) = \omega(\tau), \quad \varphi_2(t_f) = T_f. \tag{10}$$

The solution of the temperature problem in Eqs. (7) and (8) is found in implicit form.

A differential equation may be obtained from Eq. (7)

$$k(\sigma_b(A_1^4 - \omega^4) + \alpha(A_1 - \omega))\, dt = Rc\rho\, d\omega; \quad k' = \frac{k}{Rc\rho}.$$

In dimensionless form

$$\frac{\mathrm{Sk}}{k}\, d\,\mathrm{Fo} = \frac{d\theta}{1 + \frac{\mathrm{Bi}}{\mathrm{Sk}} - \frac{\mathrm{Bi}}{\mathrm{Sk}}\theta - \theta^4},$$

where

$$\mathrm{Fo} = \frac{\lambda}{c\rho}\frac{t}{R^2}, \quad \theta = \frac{\omega}{A_1}, \quad \mathrm{Bi} = \frac{\alpha R}{\lambda}, \quad \mathrm{Sk} = \frac{\sigma A_1^3 R}{\lambda}.$$

Integration with respect to Fo from $\mathrm{Fo}_1 = \mathrm{Fo}(0)$ to $\mathrm{Fo}_2 = \mathrm{Fo}(\tau)$ and with respect to θ from $\theta_1 = \theta(T_0)$ to $\theta_2 = \theta(\omega)$ gives

$$\frac{1}{k}(\mathrm{Fo}_2 - \mathrm{Fo}_1) = -\frac{1}{\mathrm{Sk}}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta^4 + \frac{\mathrm{Bi}}{\mathrm{Sk}}\theta - \left(1 + \frac{\mathrm{Bi}}{\mathrm{Sk}}\right)}.$$

Note that [2]

$$\int \frac{d\theta}{\theta^4 + \frac{\mathrm{Bi}}{\mathrm{Sk}}\theta - \left(1 + \frac{\mathrm{Bi}}{\mathrm{Sk}}\right)} = -\left[M \ln \frac{\theta^2 + a_2\theta + b_2}{-\theta^2 - a_1\theta - b_1} + \right.$$

$$+N\ln\frac{\theta-b_1}{1-\theta}+L\operatorname{arctg}\frac{\sqrt{2}}{2}\frac{2\theta+a_2}{\sqrt{\alpha_0+\alpha_1}}\Bigg]+C,$$

where C is an arbitrary constant;

$$M=\frac{\sqrt{2\alpha_0}}{\alpha_1^2+\sigma\alpha_0^2};\quad N=\frac{\alpha_1+2\alpha_0}{\left(1-\alpha_0+\dfrac{\alpha_1}{2}\right)(\alpha_1^2+2\alpha_0^2)};$$

$$L=\frac{\sqrt{2}(\alpha_1-2\alpha_0)}{\sqrt{\alpha_0+\alpha_1(\alpha_1^2+\sigma\alpha_0^2)}};$$

$$\alpha_0=\sqrt[3]{\frac{p^2}{16}+\sqrt{\frac{p^4}{16^2}-\frac{q^3}{27}}}+\sqrt[3]{\frac{p^2}{16}-\sqrt{\frac{p^4}{16^2}-\frac{q^3}{27}}}.$$

If $\overline{R}$ (θ, Bi, Sk) is the original integrand, the dimensionless body temperature θ at time Fo_2 is determined as the root of the equation

$$\frac{1}{k}(\mathrm{Fo}_2-\mathrm{Fo}_1)=-\frac{1}{\mathrm{Sk}}[\overline{R}(\theta_2,\ \mathrm{Bi},\ \mathrm{Sk})-\overline{R}(\theta_1,\ \mathrm{Bi},\ \mathrm{Sk})]. \tag{11}$$

Equation (11) is the solution of Eqs. (7) and (8) in implicit form.

Finding θ_2, it is possible to determine

$$\omega(\tau)=\theta_2A_1. \tag{12}$$

Proceeding analogously for Eqs. (9) and (10), the following equation is obtained at time t_f in dimensionless form

$$\frac{1}{k}(\mathrm{Fo}_f-\mathrm{Fo}_2)=-\frac{1}{\mathrm{Sk}'}[R'(\theta_k',\ \mathrm{Bi},\ \mathrm{Sk}')-R'(\theta_2',\ \mathrm{Bi},\ \mathrm{Sk}')]. \tag{13}$$

Adding Eqs. (11) and (13), taking into account that $Fo_1 = 0$, and using the previous notation, an equation for determining $\omega(t_2)$ from Eq. (12) is obtained

$$\frac{1}{k}\frac{\lambda t_f}{\rho R}=-\frac{\lambda}{\sigma A_1^3}\left[\overline{R}\left(\frac{\omega(\tau)}{A_1},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_1^3R}{\lambda}\right)-\overline{R}\left(\frac{T_0}{A_1},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_1^3R}{\lambda}\right)\right]-$$
$$-\frac{\lambda}{\sigma A_2^3}\left[R'\left(\frac{T_f}{A_2},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_2^3R}{\lambda}\right)-R'\left(\frac{\omega(\tau)}{A_2},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_2^3R}{\lambda}\right)\right]. \tag{14}$$

Solving Eq. (14) for $\omega(\tau)$ on the segment $[T_0, A_1]$, $\omega(t_2)$ is obtained. The time t_2 is defined as

$$t_2=\mathrm{Fo}_2\frac{c\rho R^2}{\lambda}, \tag{15}$$

where

$$\mathrm{Fo}_2=-\frac{\lambda}{\sigma A_1^3R}\left[\overline{R}\left(\frac{\omega(t_2)}{A_1},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_1^3R}{\lambda}\right)-\overline{R}\left(\frac{T_0}{A_1},\ \frac{\alpha R}{\lambda},\ \frac{\sigma A_1R}{\lambda}\right)\right].$$

Numerical calculations whow that $\tau \to +\infty$ as $\omega(\tau) \to A_1$. On the other hand, $\omega(t_2)$ tends to A_1, which leads to considerable computational error in determining t_2 on the right-hand side of Eq. (14).

Thus, the sequence of finding $\omega(t_2)$ and t_2 to determine $\varphi_2(t)$ from Eqs. (9) and (10) is unsatisfactory, because of the large computational error.

Another method of finding $\omega(t_2)$ and t_2 is possible. Basically, the time t_2 is found first, and then the temperature $\omega(t_2)$. The algorithm may be written in the following form.

Step 1. Specify the accuracy of the calculation ε and set $a := 0$, $b := t_f$.

Step 2. Calculate $\tau := (a + b)/2$.

Step 3. Integrating Eq. (7) with the initial condition in Eq. (8), find $\omega(\tau)$.

Step 4. Integrating Eq. (9) with the initial condition $\varphi_2(\tau) = \omega(\tau)$, find $\varphi_2(t_f)$.

Step 5. If $\varphi_2(t_f) > T_f + \varepsilon$, then b: = τ; proceed to Step 2.

Step 6. If $\varphi_2(t_f) < T_f - \varepsilon$, then a: = τ; proceed to Step 2.

Step 7. If $|\varphi_2(t_f) - T_f| \leq \varepsilon$, set $t_2 = (a + b)/2$; the calculation is over.

NOTATION

t, time; T(t), temperature of metal at time t; $T_c(t)$, temperature of medium at time t; k', reduced form factor of body; α, convective thermoemission coefficient; σ, radiative thermoemission coefficient; T_0 and T_f, initial and final temperature of metal; t_f, fixed duration of heating process; β, ratio of activation energy to gas constant; κ, specified constant characterizing the dynamics of skin growth; A_1, A_2, minimum and maximum temperature of medium; k, form factor of body; R, characteristic dimension of body; c, specific heat; ρ, density of material.

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HEAT TRANSFER ON MOUNTING ELECTRONIC COMPONENTS ONTO A PRINTED CIRCUIT BOARD

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The results of mathematical modeling of nonlinear nonsteady heat-transfer processes in conditions of IR soldering of components onto printed circuit boards are outlined.

Establishing the thermophysical principles of the technological process of mounting electronic components on a printed circuit board (PB) is an urgent problem in the development of this technology [1-3]. Mathematical modeling plays an important role in solving this problem, because of its many well-known advantages: the possibility of compensating for the unavoidable omissions of experimental work, significant reduction in the volume of expensive full-scale experiments, the simplicity of investigating different parameter combinations, etc.

The aim of the present work is the mathematical modeling of heat transfer in the technology of surface mounting. The physical situation corresponding to the use of resistive and IR heating is analyzed. The temperature conditions of printed elements (PE) corresponding to all the possible single electronic-engineering components (EEC) which may be mounted on PB of highly and poorly heat-conducting materials are studied. Thus, outputless EEC (a monolithic tantalum capacitor and a matrix microframe) and EEC with planar outputs (an N-type microframe and a plastic frame) are considered, in the process of mounting on ceramic and glass-textolite PB.

In characterizing the processes that occur overall in the given conditions, a series of factors complicating the investigation may be noted: in particular, nonlinearities of various kinds, complex configurations of the given regions, phase transitions (melting and hardening of the solder), PE Motion in the furnace for IR soldering, diversity of the PE constructional materials, etc. Consequently, the preliminary investigations must include

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